Journal of Applied Mechanics and Technical Physics, Vol. 36, No. 5, 1995

DYNAMIC THERMOSTABILITY OF THE COMPOSITE SHELLS OF A SANDWICH-TYPE STRUCTURE

Yu. V. Nemirovskii, V. I. Samsonov, and A. V. Shulgin

UDC 539.3 : 678.06

Modern materials are complex in their composition and also in the variety of structure, phase state, etc. The conventional "classical" methods for calculating the constructionl elements of such composite materials (CM) can lead, in some cases, to substantial errors in the basic calculated characteristics. Thus, the development of new refining mathematical models and of calculation techniques (mainly numerical) seems very urgent, especially for considering the problems on the dynamics of constructions in nonlinear settings of boundary-value problems.

This paper generalizes the statement of the nonlinear model of [1] to the case of thermoelasticity of a sandwich-type shell with regard to the interaction between layers with different physical characteristics. A version of the advanced calculation model of thermoelastic deformation of sandwich-type CM-shells (including those suitable for calculating both smooth and reinforced shells) has been constructed taking into account the interactions of layers continuous in tangential forces and the shift vector, the interlayer shifts along the packet height as a whole, and also the existence of transient zones between the matrix and the armoring fibers arising from production processes. The specificity of a composite structure is reflected in the elasticity coefficients of the system, which naturally involve the physical and mechanical characteristics of individual material substructures. The problems on the nonlinear thermoelastic deformation of a sandwich-type cylindrical composite shell under the action of power and temperature pulses are realized numerically. The obtained data are analyzed, and practical recommendations on their use are given.

The decision equations of nonlinear bending, stability, and vibrations of sloping sandwich-type composite shells were obtained in [1]. However, when it is necessary to consider the thermoelastic statements of problems, one should introduce additional components using the Duhamel–Neuman relations generalized for the case of thermoelasticity. For the variant of anisotropy with only one plane of elastic symmetry, these can be written as follows:

$$\sigma_k = \sum_{l=1}^{3} A_{kl}\varepsilon_l - \beta_k^T \vartheta, \quad \sigma_{i3} = \sum_{j=1}^{2} G_{ij}\varepsilon_{j3} - \beta_{i3}^T\vartheta \quad (k = 1, 2, 3, \quad i = 1, 2). \tag{1}$$

where the first terms are taken from [1]. In this case, $\sigma_3 \equiv \sigma_{12}$, $\varepsilon_3 = \varepsilon_{12}$, and the coefficients of the thermal effect are of the form

$$\beta_k^T = \sum_{l=1}^{3} A_{kl}\alpha_l^T, \qquad \beta_{i3}^T = \sum_{j=1}^{2} G_{ij}\alpha_{j3}^T, \tag{2}$$

where $\vartheta = T - T_0$; T is the absolute temperature of the body; T_0 is its initial component; and α_l^T, α_{j3}^T are the coefficients of the thermal expansion of the constructional material [2]. Expressions (1) and (2) can be written for each of the shell layers identically, and therefore, the indices referring to the layers are omitted.

Using (1) and (2) and carrying out a rather cumbersome procedure, similar to [1], to derive the decision equations, we obtain a decision system for the thermoelastic deformation of the sandwich-type CM-shells in

Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 36, No. 5, pp. 164–172, September–October, 1995. Original article submitted October 17, 1994.

0021-8944/95/3605-0781 $12.50

the generalized shifts:

$$\sum_{m=1}^{2}(\Delta_{im}u_m+\Delta'_{im}\alpha_m)-J_iw-U_i^*-\varepsilon^{-1}\underline{\nabla_i\vartheta^*}+\varepsilon^{-2}q_{i3}=0,$$

$$\sum_{m=1}^{2}(\Delta'_{mi}u_m+\Delta''_{im}\alpha_m)-J'_iw-K_i^*-\varepsilon^{-1}\underline{\nabla'_i\vartheta^*}+\varepsilon^{-2}q_{i3}^0=0, \quad (3)$$

$$\sum_{m=1}^{2}(J_m^0u_m+J_m'^0\alpha_m)-J_3w-L_3^*-\underline{\nabla_3\vartheta^*}-\varepsilon^{-2}q=0, \quad \mathbf{Q}\delta\mathbf{u}\Big|_\Gamma=0 \qquad (i=1,2).$$

In this case, u_i, w are the main shifts on the reference surface; α_i are the coordinate and time functions related to transverse shifts; Δ_{im}, Δ'_{im}, Δ''_{im}, etc. are the differential operators in partial derivatives; U_i^*, K_i^*, and L_3^* are the inertial components; q_{i3}, q_{i3}^0, and q are the parameters of the external surface loading; and $\mathbf{Q}$ and $\delta\mathbf{u}$ are, respectively, the vectors of generalized forces and variations of the generalized shifts along the Γ-line bounding the reference shell surface. The influence of interlayer shifts and the interaction between the fibers and the binder are taken into account. The conditions for their contact in the packet not only upon shifts but also by tangential forces are made consistent [3–5]. The terms underlined correspond to the temperature components $\vartheta^*=\alpha^T\vartheta$ (α^T is the characteristic coefficient of thermal expansion). The operators ∇_i, ∇'_i, and ∇_3 are linear and contain the elastic and thermal (in the general case) components of the transverse coordinate. Since the operators involved in (3) are too cumbersome, they are neglected and, if necessary, will be written in solving particular problems. Note also that the order of the decision system of equations (3) does not depend on the number of layers and their arrangement.

We use system (3) to study the buckling of the cylindrical composite three-layer shell under periodic dynamic actions (force and temperature). To this end, the system of stability equations deduced from (3) will be supplemented with both the generalized heat equation and the initial conditions. It is noteworthy that when solving problems of stability subject to boundary conditions one must neglect temperature terms [6] because they will be taken into account in determining the subcritical state of the shell and the equations of supercritical deformation will preserve the form similar to that in [3, 4, 7] irrespective of the existence of the temperature component.

The resulting complete problem on dynamic thermostability has a fairly complex form and structure. Its direct solution is difficult even by numerical methods. Thus, according to the remark in [8] on approaches to the solution of such problems, we consider now two steps of their realization. In the first step, it is assumed that the change in the thermoelastic deformation along the shell is independent of bending and can be determined from the solution of the corresponding one-dimensional problem, as for an elastic rod. In the second step, we solve the problem on nonlinear supercritical deformation (buckling) under the action of temperature and force factors. In this case, the temperature components will be involved in the system of equations as the well-known values determined in the previous step.

The generalized heat equation for an anisotropic body (the composite material of the shell) will be derived from the relation [2]

$$T\frac{\partial S}{\partial t}=-q_{i,i}, \quad (4)$$

where $\mathbf{q}$ is the heat flux vector and S is the entropy per unit volume. In terms of the Fourier law for an anisotropic medium: the vector

$$q_i=-\lambda_{ij}T_{,j}. \quad (5)$$

Since we are interested in the one-dimensional case of propagation of heat deformations, the right-hand side of Eq. (4) reduces to

$$q_1=-\lambda T_{,1} \quad (6)$$

(λ is the heat conductivity in the longitudinal direction of the shell). The entropy in the anisotropic case is of the form

$$S = \beta_{ij}\varepsilon_{ij} + (C_\varepsilon \rho / T_0)\vartheta. \tag{7}$$

Here C_ε is the specific heat at constant strain; ρ is the material density; β_{ij} are the material constants related to the mechanical and thermal properties of the material, which are determined below; and ε_{ij} is the strain tensor. The first term in (7) for entropy characterizes the conjugation of the deformation field with temperature and the second one is typical of the entropy caused by heat conductivity. Substituting (7) into (4) and taking into account (6), we obtain

$$T_0\beta_{ij}\varepsilon_{ij} + C_\varepsilon \rho \vartheta = \lambda \frac{\partial^2 \vartheta}{\partial x_1^2} \tag{8}$$

(x_1 is the longitudinal cylindrical coordinate). Taking into account the one-dimensionality of this case and the expression for strains in terms of shifts according to [1] we convolve the first term in (8). As a result, relation (8) in terms of dimensionless variables will take the form

$$\frac{\partial \vartheta^*}{\partial \tau} - \Lambda \frac{\partial^2 \vartheta^*}{\partial \xi_1^2} + \Omega \frac{\partial^2 u_1}{\partial \tau \partial \xi_1} = 0, \tag{9}$$

where $\vartheta^* = \alpha^T \vartheta$; $\tau = ct/L$; $c = \sqrt{E_0/\rho}$; $\Lambda = \lambda L / R^2 C_\varepsilon \rho c$; t is the dimensional time; $\Omega = \varepsilon A_{11}^* T_0 (\alpha^T)^2 E_0 / C_\varepsilon \rho$; $\xi_1 = x_1/R$; L, R are the length and mean radius of the cylindrical surface of the shell; E_0 is the elasticity modulus of the isotropic binder of the material; ρ is the mean density of the shell material; u_1 is the displacement of the points of the middle surface toward the element, normalized by the overall thickness of the shell; $\varepsilon = H/R$: and A_{11}^* is the elastic CM modulus along the element [1]. Equation (9) differs from the conventional heat equation by its additional term related to work of deformation, i.e., we have a so-called bound or generalized heat equation, which should be considered together with the equation of motion (3) at $i = 1$. Reducing it to dimensionless form and using the above designations, we obtain

$$\frac{\partial^2 u_1}{\partial \xi_1^2} - \varepsilon^{-1} \frac{\partial \vartheta^*}{\partial \xi_1} - K \frac{\partial^2 u_1}{\partial \tau^2} = 0 \tag{10}$$

($K = R^2 / L^2 A_{11}^*$). Equations (9) and (10) form a complete system of differential equations of the bound thermoelasticity of a composite cylindrical shell in a one-dimensional variant. The boundary and initial conditions for the case under study are of the form

$$\frac{\partial \vartheta^*}{\partial \xi_1} = u_1 = 0 \quad \text{at} \quad \xi_1 = 0, \frac{L}{R}, \quad \frac{\partial u}{\partial \tau} = u_1 = 0 \quad \text{at} \quad \tau = 0, \quad \vartheta^* = \vartheta_0^* \quad \text{at} \quad \tau = 0, \ \xi_1 = 0. \tag{11}$$

The last condition in (11) corresponds to the application of a heat pulse ϑ_0^* to one of the shell faces at the initial moment, i.e., dramatic local heating of the remaining heat-insulated surface [8]. Solving the main problem on the buckling of a composite sandwich-type shell, one should introduce the temperature effect component into the decision system of stability equations [7] according to the formula

$$T_1^0(\tau, \xi_1) = A_{11}^* \left(\varepsilon \frac{\partial u_1}{\partial \xi_1} - \vartheta^* \right). \tag{12}$$

In this case, $\partial u_1 / \partial \xi_1$, ϑ^* can be found by solving the problem (9)–(11).

Note that when the heat sources vary slowly with time the inertial component in Eq. (10) can be neglected. Hence

$$\frac{\partial}{\partial \xi_1} \left[\varepsilon \frac{\partial u_1}{\partial \xi_1} - \vartheta^* \right] = 0. \tag{13}$$

System (9), (11), (13) now forms a so-called quasistatic thermoelasticity problem. Equation (13) can readily be integrated for the boundary conditions $u_1(0, \tau) = u_1(L/R, \tau) = 0$. As a result, $T_1^0(\tau)$ can be written in the explicit form

$$T_1^0(\tau) = -A_{11}^* \frac{R}{L} \int\limits_0^{L/R} \vartheta(\tau, \xi_1)\, d\xi_1, \tag{14}$$

where the function $\vartheta(\tau, \xi_1)$ is determined by solving the problem (9) and (11) on generalized heat conductivity. This is especially evident when passing to a new time variable $\tau_1 = \Lambda\tau$ in (9) and (10). Hence, Eq. (10) takes the form

$$\frac{\partial^2 u_1}{\partial \xi_1^2} - \varepsilon^{-1}\frac{\partial \vartheta^*}{\partial \xi_1} - K\Lambda^2 \frac{\partial^2 u_1}{\partial \tau_1^2} = 0. \tag{10a}$$

For actual materials the coefficient $K\Lambda^2$ for the second time derivative is so small that the last term in (10a) can be neglected compared to the remaining. We again arrive at (13).

Thus, the problem solved in the first step reduces to the integration of the system of equations (9) and (10) subject to the initial boundary conditions (11).

As mentioned above, the problem on thermoelastic buckling with boundary conditions will contain no heat additives because the latter have already been taken into account in solving the problem in the first step. In this case, the form of the stability equations given in [3, 4, 7] is preserved, preserve their form and only changes due to the existence of the temperature component must be introduced. Solving again the initial set of stability equations proposed in [3, 4] and performing cumbersome transformations, we obtain a nonlinear system of ordinary differential equations for the dimensionless amplitude values ζ_1 and ζ_2 (referred to the full coating thickness H):

$$\begin{gathered} \frac{1}{S}\frac{d^2\zeta_1}{dt_*^2} + \zeta_1(1 + T_1^0 a_{01} - a_0 t_*) - a_1\zeta_1\zeta_2 + a_2\zeta_1\zeta_2^2 + a_3\zeta_1^3 = 0, \\ \frac{6\varepsilon^{-1}}{S}\frac{d^2\zeta_2}{dt_*^2} + \zeta_2(a_4 + 2a_{01}T_1^0 - a_5 t_*) - a_6\zeta_2^2 - a_7\zeta_1^2 + a_8\zeta_2\zeta_1^2 + a_9\zeta_2^3 = 0. \end{gathered} \tag{15}$$

In this case all the values are dimensionless and a_0, a_{01}, ..., a_9 are the coefficients containing the physical and geometrical parameters of the shell:

$$\begin{gathered} a_0 = \lambda_0/\lambda_{\rm cr}, \quad a_{01} = \eta^2 M_{01}/n^2 M, \quad a_1 = \varepsilon(M_3 + M_1/2)/M, \quad a_2 = (m_{53}M_{01} + \varepsilon M_4)/M, \\ a_3 = (\varepsilon^2/32)m_{52}M_{01}/M, \quad a_4 = M_{06}M_{01}/M, \quad a_5 = 2\eta^2 t_1^0 \lambda_0 M_{01}/n^2 M, \quad a_6 = M_{66}M_{01}/M, \quad a_7 = M_2/M, \\ a_8 = (\varepsilon^2/4)[M_5 + (A_{11}^*\eta^4 + 3A_{12}^*\eta^2)M_{01}]/M, \quad a_9 = (\varepsilon^2/4)A_{11}^*\eta^4 M_{01}/M, \quad \lambda_{\rm cr} = n^2 M/(t_1^0\eta^2 + t_2^0)M_{01} \end{gathered} \tag{16}$$

(the components of these coefficients have a fairly cumbersome structure and thus are omitted; M_s, M_{01}, M, etc. are the determinants of the kinematic matrix-operator using the Bubnov–Galerkin procedure [3, 4]); $t_* = \lambda/\lambda_0$; $\lambda = s_1 t/E_0$ is the dimensionless parameter of the force loading; s_1 is the increase in the velocity of loading; t is the dimensional time; λ_0 is the minimal upper critical loading parameter, obtained from the static linear problem; t_1^0, t_2^0 are the dimensionless parameters of the external force action in the axial and lateral directions, respectively; $\eta = m\pi R/nL$ (m, n being the integers of wave formation in the longitudinal and circumferential directions); and $S = (cE_0/s_1 R)^2 n^4 \lambda_0^2 M/M_{01}$.

In the quasistatic loading of the shell by external forces, the inertial terms in Eqs. (15) must be assumed zero. Hence

$$a_0\lambda/\lambda_0 = 1 + T_1^0 a_{01} - a_1\zeta_2 + a_2\zeta_2^2 + a_3\zeta_1^2, \quad a_5\lambda/\lambda_0 = a_4 + 2a_{01}T_1^0 - a_6\zeta_2 - a_7\zeta_1^2/\zeta_2 + a_8\zeta_1^2 + a_9\zeta_2^2. \tag{17}$$

Expressing ζ_2 from the second equation of (17) in terms of ζ_1 and taking into account only the first

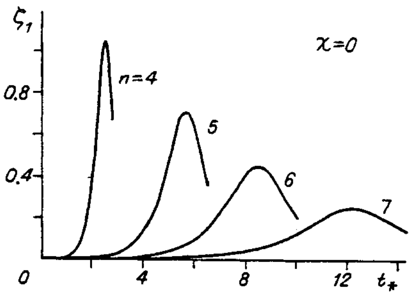

Fig. 1

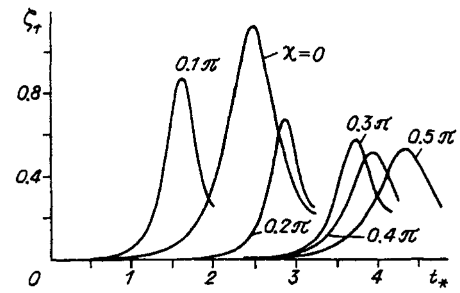

Fig. 2

TABLE 1

Layer No.	Variant					
	1	2	3	4	5	6
	Rigid characteristics of layers in four directions					
1	$0, 0, \Omega_0/2, \Omega_0/2$	$\Omega_0, 0, 0, 0$	$\Omega_0, 0, 0, 0$	$0, \Omega_0, 0, 0$	$0, 0, \Omega_0/2, \Omega_0/2$	$0, 0, \Omega_0/2, \Omega_0/2$
2	$0, \Omega_0, 0, 0$	$0, 0, \Omega_0/2, \Omega_0/2$	$0, \Omega_0, 0, 0$	$\Omega_0, 0, 0, 0$	$0, 0, \Omega_0/2, \Omega_0/2$	$0, 0, \Omega_0/2, \Omega_0/2$
3	$\Omega_0, 0, 0, 0$	$0, \Omega_0, 0, 0$	$0, 0, \Omega_0/2, \Omega_0/2$	$0, 0, \Omega_0/2, \Omega_0/2$	$\Omega_0, 0, 0, 0$	$0, 0, \Omega_0/2, \Omega_0/2$

power of ζ_2, we get the dependence $\lambda/\lambda_0 = f(\zeta_1)$ in the form

$$a_0\frac{\lambda}{\lambda_0} = 1 + T_1^0 a_{01} - \frac{a_1 a_7 \zeta_1^2}{a_4 + a_8\zeta_1^2} + \frac{a_2 a_7^2 \zeta_1^4}{(a_4 + a_8\zeta_1^2)^2} + a_3\zeta_1^2. \tag{18}$$

Relation (18) describes the postcritical behavior of the shell for different wave numbers m and n and for given physical and geometrical parameters. Constructing the curves $f(\zeta_1)$, we determine the so-called "lower" critical load and the corresponding branch of the supercritical shell strain.

In solving many problems on dynamic instability [8] the value of ζ_2 determined from (17) has been used for system (15). In this case we get one equation for ζ_1:

$$\frac{1}{S}\frac{d^2\zeta_1}{dt_*^2} + \zeta_1(1 + T_1^0 a_{01} - a_0 t_*) - \frac{a_1 a_7 \zeta_1^3}{a_4 + a_8\zeta_1^2} + \frac{a_2 a_7^2 \zeta_1^5}{(a_4 + a_8\zeta_1^2)^2} + a_3\zeta_1^3 = 0 \tag{19}$$

(the initial conditions are given for ζ_1 only).

Thus, in the second step of solution of the problem on thermostability, we have system (15) [or only Eq. (19)] with initial conditions for ζ_1, ζ_2, and $\dot{\zeta}_1$, $\dot{\zeta}_2$ (or ζ_1, $\dot{\zeta}_1$) at $t_* = 0$ and hence the problem on shell buckling under dynamic, temperature, and force loadings is formulated.

In studying the process of buckling we assume that the shell becomes unstable when the amplitude of maximum deflection dramatically increases with increasing t_* for the near-ordinate values of n (parameter η is fixed). It is also believed that the critical dynamic loading is achieved if the arrow of maximum deflection is assumed to exceed the static deflection corresponding to the highest value of the external action [8]. Some authors (see, e.g., [9]) consider the loading critical if the stress intensity reaches the limit of proportionality. All these criteria are conditional, display no universal character, and refer to specific shells with definite mechanical and geometrical parameters. Therefore, in this case, as a criterion for instability, we take the

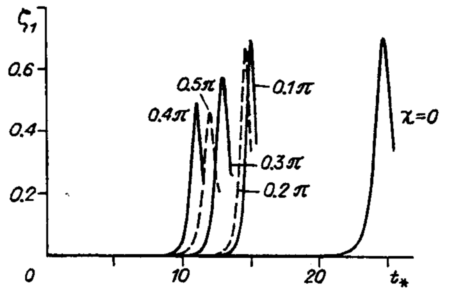

Fig. 3

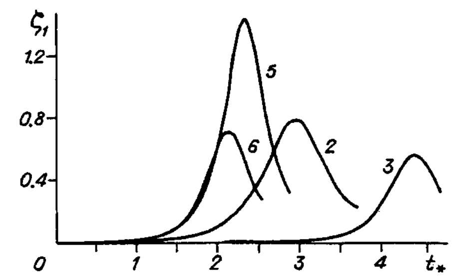

Fig. 4

former as the most obvious, at which the so-called "skipping" of the shell occurs and further dynamic loading quickly leads to exhaustion of its carrying ability.

Solving the problem on the dynamic thermoelastic behavior of a three-layer composite shell for system (15) [or in other cases for Eq. (19)] with the initial conditions $\zeta_1 = \zeta_2 = 0.001$ and $\dot{\zeta}_1 = \dot{\zeta}_2 = 0$ (or $\zeta_1 = 0.001$, $\dot{\zeta}_1 = 0$) with $t_* = 0$, we used Gear's scheme [10] dealing with the so-called "rigid" equation in which the coefficients differ from one another by orders of magnitude. System (9), (10) for the initial boundary conditions (11) was integrated by the finite-difference method. In cases allowing certain simplifications, we used relation (14). In the calculations the structure of a sandwich-type shell obeys the condition [3]

$$\sum_{s=1}^{3} h_{0s}\Omega_{0s} = \Omega_0, \tag{20}$$

where $h_{0s} = h_s/H$; $\Omega_{0s} = \omega_a^{(s)}E/E_0$; $\Omega_0 = c_0E/E_0$; $\omega_a^{(s)} = \sum\limits_{k=1}^{4}\omega_k^{(s)}$ is the volume content of reinforcing elements in the sth layer; and c_0 is the total volume content of reinforcement in the shell packet. Condition (20) makes it possible to vary the structural parameters in the layers to choose the best design of a three-layer shell for the given external pulses of force and temperature action. Thus, Fig. 1 shows the characteristic dependences $\zeta_1(t_*)$ under the action of the temperature pulse $\theta_0^* = 0.3$ for different rigid parameters in the layers (variant 3 in Table 1 giving the reinforcing intensities in four directions), for relative thicknesses $h_{01} = h_{02} = 0.25$, $h_{03} = 0.5$, and for different numbers of circumferential wave formation n ($\Omega_0 = 96$, $\chi^{(3)} = 0$). It is obvious that the behavior of the shell is characterized by increase in the dynamic coefficient t_* with increasing n and decreasing amplitude of skipping ζ_1 upon transition to a new state. In this case, for $n = 4$ the "skipping" process starts earlier than for other values.

Simultaneously, as an intermediate result, the dimensionless $u_1(\tau)$ and $T_1^0(\tau)$ were calculated in the middle of the three-layer composite shell ($\xi_1 = L/2R$) to obtain a pattern of perturbation wave propagation with time. $T_1^0(\tau)$ was also determined from formula (14). The data obtained were put in the basic equation (19) of dynamic instability. The calculated results indicate that the $\zeta_1(t_*)$ dependences essentially coincided with the introduction of parameter $T^*(\tau)$ calculated by the approximate and exact formulas. The only difference was a negligible decrease in the vibration amplitude after "skipping" of the shell, and the beginning of transition of the shell to another state coincided completely.

A similar pattern of the behavior of the dependence $\zeta_1(t_*)$ was also observed for the other reinforcing variants and therefore in Figs. 2–4 $n = 4$ was assumed and the other parameters were varied. In particular, under the joint action of the temperature pulse and of the force pulse of intensity $s_1 = 98 \cdot 10^7$ Pa/sec, Fig. 2 (reinforcing variant 4, see Table 1) shows the dependences $\zeta_1(t_*)$ with varying $\chi^{(3)}$ angle of the directional packing of reinforcing elements in the middle layer. Evidently, the largest dynamic coefficient (parameter t_*) will be observed with $\chi^{(3)} = 0.5\pi$. A similar structure with $\chi^{(3)} = 0.4\pi$ provides the largest static critical loading obtained by linear theory [1]. Comparing the curves for $\chi^{(3)} = 0.1\pi$ and $\chi^{(3)} = 0.5\pi$, we see that, varying the angle of the directional packing of reinforcing elements, we can substantially increase the dynamic coefficient with the other parameters being constant (reinforcing intensities and their ratios in layers). Of great interest are the results given in Fig. 3. In this case, instead of the positive thermal pulse, a negative thermal pulse is applied accompanied by the action of a force pulse of the same intensity as shown in Fig. 2. As a result, the reverse effect is observed, i.e., the largest dynamic coefficient is recorded for $\chi^{(3)} = 0$, and the smallest one, for $\chi^{(3)} = 0.4\pi$. This calculation variant resembles the effect of "weak" rigidity under the action of only force dynamic loading of the shell [3, 7], i.e., the less rigid the shell in the circumferential direction (variant 4, see Table 1). the higher its dynamic coefficient.

Figure 4 gives the values of $\zeta_1(t_*)$ for a given angle of the directional packing of reinforcing elements in the layers $\chi = 0.4\pi$ of the parameter $n = 4$, at which in most cases the first dramatic increase of deflections begins, and for different reinforcing schemes in Table 1 (the figures near the lines correspond to the variants in Table 1) as the shell is loaded simultaneously with positive thermal and force pulses. It is evident that a more rational scheme in this case is variant 3, which is the best in static loading [1] and also under the action of only force dynamic loading [7]. Recall that the dynamic coefficient is determined as the ratio of the current loading to the corresponding static value obtained by linear theory [8]. Although rather relative, the characteristic fairly adequately reflects the behavior of the sandwich-type CM-shell in absolute values, i.e., upon transition to dimensional values.

As was mentioned in [11]. a "weak point" in the dynamic description of the behavior of shells is generally the fact that solutions are sought for particular projects rather than for the whole class. However, in practice, the calculation is always performed for particular constructional elements and for given types of loading. Thus, the so-called "weak point" is justified by obtaining reliable results for the given criteria of stability. One need not strive for generalizations, which in each specific case must be verified by particular projects.

Finally, note that the results obtained illustrate the approach developed for determining the dynamic characteristics of CM-shells of sandwich-type structure in different thermoforce loading regimes using the geometrically nonlinear theory of bending. For a more complex rheology, one should only introduce changes in the determinants of the kinematic matrix operators M_{0s} and make use of the approach proposed for solving the dynamic problem on nonlinear deformation. In this regard, the above examples illustrate the efficiency of the application of the precise model proposed in this work. At the same time, it can be employed for a wider class of thin-walled constructions used in different areas.

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